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Forced Response of a Vortex Filament Pair Measured in a Water Tunnel

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Crow instability is examined and quantitative measurements made in a water tunnel are presented. The results of the application of a novel technique for quantitative data acquisition that allows measurement of the vortex strength, the oscillation frequency of the vortex filaments, and the rate of growth of their amplitude simultaneously, are presented. A pair of vortex filaments, typical of those of wing tips, is created by individual vortex generating blades in a water tunnel. The filaments are then subjected to periodic inputs, and their responses are measured downstream. The vortex system is excited over a range of frequencies. The filaments are visualized with dye injection, whereas their cross sections are illuminated with light sheets at different downstream locations. Time history of the vortex position is extracted from video recordings of the illuminated cross sections. The results show that most of the essential features associated with Crow instability, such as the inclination angle of the plane of vortex motion and the growth rate of the amplitude, are in qualitative agreement with the theoretical model. Disagreements between theoretical and experimental results are examined and discussed. These experiments prove the viability of the experimental approach. Furthermore, the results demonstrate clearly that manipulation of the excitation frequency can control the rate of growth of amplitude.

Nomenclature

a	=	amplitude growth rate from e^{at}
b	=	separation between vortices, vortex span
C	=	chord length
f	=	frequency, Hz
k	=	wave number, ω/V_∞
t	=	time
V_∞	=	freestream velocity
w	=	downwash velocity
x	=	streamwise axis, measured from the blade's trailing edge
y	=	horizontal axis normal to the freestream
z	=	vertical axis normal to the freestream
α	=	nondimensional amplitude growth rate, $(2\pi b^2/\lvert\Gamma\rvert)a$
β	=	nondimensional wave number, kb
ΔT	=	water tunnel temperature minus mixture temperature
δ	=	nondimensional integration cutoff distance
θ	=	principal axis inclination angle
Γ	=	vortex strength
ω	=	shaking frequency, rad/s

Introduction

FOR the last three decades, vortices have been the subject of mathematical analysis, experimental work, and numerical simulation. Such authors as Andrews,[1] Andrews et al.,[2–5] Crow,[6] Parks,[7] and Rossow,[8] among others, have pioneered the study of vortex dynamics. Of particular interest is the behavior of a pair of vortices resulting from the roll up of the wake generated by a lifting wing, commonly called wingtip vortices. The behavior of these vortices has been the subject of intense research for several decades, from two different viewpoints.

The first consequence of wingtip vortices is the formation of induced drag. Reduction of this quantity has led to a variety of schemes promoting early vortex dispersion or weakening. Some of these efforts are briefly outlined by Rokhsaz.[9] Examples include wing tip turbines[10,11] to extract energy from the wingtip vortex and splines[12] placed in the wake to disperse the wingtip vortex. Other devices, such as winglets,[13] wingtip sails,[14,15] and ogee tips[16] have been tested with varying degrees of success.

Aircraft wake turbulence has been the second major issue associated with wingtip vortices. These pairs of counter-rotating vortices are notoriously persistent and require separation distances between aircraft on the order of several miles. These separation distances, adopted by various aviation regulatory establishments throughout the world, constitute a major obstacle to increasing airport traffic while maintaining acceptable levels of safety. A comprehensive bibliography of research on aircraft wake vortex hazard is presented by Hallock.[17] Among the efforts cited by Hallock are a large number of flight tests that show the strong influence of wake vortices on following aircraft.[1–5] These tests highlight the longevity of the vortices and show that viscous dissipation alone is too lengthy a process to render the vortices safe for penetration by a following aircraft in a reasonable distance.

A prominent feature of the flow for a pair of counter-rotating vortices is the long wavelength Crow[6] instability, eventually yielding the decay of the vortex pair. In his landmark paper, Crow[6] presented a linear model of the vortex pair, simplified to the first order in radial displacement. This model showed that the mutual interaction between two vortex filaments could result in unstable stationary waves whose amplitudes vary as an exponential function of time e^{at} and whose wavelength is typically eight times the vortex separation distance. This model predicted that these stationary waves are confined to planes inclined at approximately 45 deg with respect to the plane of the vortices. His study also showed that there are two modes of instability, symmetric and antisymmetric. Whereas the antisymmetric mode seems to be very rare, the symmetric mode is

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a fairly frequent occurrence and can be triggered by a pitching and plunging motion of the aircraft.[8] At long downstream distances, the symmetric mode results in the merging of the vortex filaments and the formation of rings. In this mode, the maximum growth rate a is between 0.8 and 1.2 and depends on the nondimensional frequency β. The maximum growth rate of the amplitude occurs for values of β between 0.8 and 2.

Many vortex researchers have investigated this phenomenon and the prospects for actively initiating these instabilities in a pair of trailing vortices. Individual vortices are hard to destroy because one can only rearrange vorticity and angular momentum. However, mutual induction can lead to exciting instabilities to promote vortex breakdown.[18] Efforts to accelerate the self-destruction of wingtip vortices have led to the reshaping of the spanwise lift distribution,[19] the use of vortex-generating wing fins to increase the amount of vortex interaction in the wake,[20] and the harmonic motion of control surfaces.[21] Most of these techniques have not been very practical and sometimes have entailed a drag penalty, which is not acceptable for transport aircraft.

In contrast to the abundant qualitative data on Crow instability, there have been very few experimental verifications of this phenomenon in the open literature. For the most part, Crow instability has been observed during flight tests, where experimental conditions such as the atmospheric turbulence can hardly be controlled. The rarity of experimental verification of Crow instability in a controlled laboratory environment may be attributed to four major factors.

The main requirement for observing Crow instability in a laboratory is a rather long test section. Because this instability has a long wavelength, it takes several wingspans to develop. For this reason, tow tanks have been popular among researchers such as Leweke and Williamson.[22] In a tow tank, the vortices can be observed over a long period of time, corresponding to a long distance downstream of the model. However, measurements in tow tanks can be challenging, and their dimensions are such that the vortices can approach the floor of the tank rather quickly, resulting in a strong ground effect.

Another challenge in the quantitative verification of Crow instability is measuring the amplitude of motion and its correlation with the frequency of the instability. Although it is fairly easy to excite specific frequencies in a controlled manner,[23] it is harder to measure the amplitude of motion resulting from that excitation. Leweke and Williamson[22] and Eliason et al.[23] measured the amplitude visually.

The third source of experimental challenge is the accurate measurement of the vortex strength. The primary reasons are the transient motion of the vortex filament and the viscous effects in any real fluid. A single vortex filament undergoes a self-induced type of motion that may not be harmonic, as shown by Rokhsaz et al.[24] Additionally, due to mutual induction in a flowfield containing two vortex filaments, the amplitude of motion can be larger than the viscous core diameter. In any case, the vortex motion renders evaluation of the velocity components difficult at best. Also, in the presence of viscous effects, it is not clear how to define the edge of the viscous core and, therefore, extract the vortex strength from the measured velocities.

The last factor is the impracticality of the use of wings in water or wind tunnels to generate vortices. There is an obvious tradeoff between the visibility of the vortices, and the vortex strengths needed to insure significant mutual interaction. Low speeds enable good flow visualization, but result in weak vortices and strong viscous effects. High speeds, on the other hand, create enough lift for stronger vortices, but they are typically accompanied by levels of turbulence greater than those present at low speeds.

In the present study, the authors report on their continuing efforts to study the Crow instability experimentally in a water tunnel. The current article represents the continuation of the research presented earlier by Rokhsaz et al.[25] In that paper, the authors described the development of a low-cost technique for experimental measurement of vortex motion in a water tunnel. This method relies on a combination of flow visualization and digital imagery for data acquisition. With the exception of dye injection, this method is nonintrusive because it does not interfere with the flowfield. Vortex filaments are generated by two separate wings, which allows independent control of the separation distance between the vortices and their strength. This setup is similar to those of Devenport et al.[26] and Leweke and Williamson.[22]

The results presented here include the response of the vortex pair to forced excitations with fixed amplitude. The frequency range was chosen to excite the symmetric mode of motion.

Experimental Approach

The experimental apparatus used for this research was identical to that described in Ref. 25. A brief description of the equipment and the facility is given here. However, the reader is encouraged to consult Refs. 25 and 27 for further details.

Test Facility

The work described here was performed in the water tunnel at the National Institute for Aviation Research (NIAR), located on the campus of Wichita State University (WSU). This facility is a closed-loop horizontal tunnel containing approximately 3500 gallons of water. The clear test section, visible from five directions, is 2 ft deep, 3 ft high, and 6 ft long. A 2.5-ft segment with constant cross-sectional area connects the test section to the diffuser. This section can also be used as an extension of the test section, allowing observation of a vortex filament over a length of approximately 8 ft. Water speed can be varied from 0.05 to 1.0 ft/s, corresponding to flow Reynolds numbers of 5×10^3 to 1×10^5 per foot, with very low levels of turbulence. However, the speeds of 0.2–0.6 ft/s contain the least amount of turbulence as measure by a constant temperature anomometry unit in an earlier study. Figure 1 shows the schematic of this facility.

Vortex Generation

The vortex generating system consisted of two rectangular flat blades, mounted on a reflection plane that was located approximately 4 in. below the free surface of the water. Each blade was made of 1/16-in.-thick aluminum with the chord length and the span length of 8.1 and 12 in., respectively. The blade incidence angles could be set independently for any arbitrary distance between the two vortex filaments. This arrangement allowed control of the vortex strengths and the vortex span independently. To model wing tip vortices, the two blades were set at equal and opposite angles relative to the freestream. Vortex strength could also be varied via tunnel flow speed. However, to minimize the effects of freestream turbulence, the majority of the tests were performed at a tunnel speed of 0.25 ft/s.

Blade angles of attack were varied between 6 and 8 deg, which produced a total blockage of less than 3% based on frontal area. Therefore, the blockage was deemed to be reasonably small and did not affect the measured average flow speed. Larger angles of attack

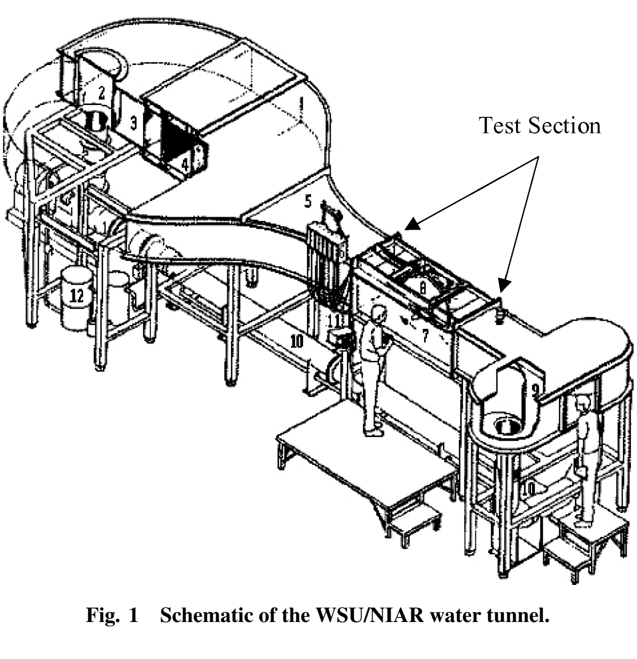


Fig. 1 Schematic of the WSU/NIAR water tunnel.

led to wakes dominated by unsteady viscous effects, whereas lower angles of attack produced very weak vortices that could not retain the dye in the cores. This led to the diffusion of the dye that made visualization very difficult, especially when the vortices were excited.

In most cases, blade separation distance was set at 3–4 in. At lower separation distances, the viscous layers of the two blades would merge, resulting in significant flow blockage between the blades. This was manifested by a noticeable spanwise flow between the blades near the tip.

Periodic Excitation

Disturbances of controlled frequency and fixed amplitude were introduced into the flow via a narrow aluminum strip that spanned the test section. This strip was mounted downstream of the trailing edges of the blades and approximately 0.75 in. above the vortex filaments. The center of the strip was connected to a shaft attached to a crank on a small dc motor. The amplitude of motion of the strip was fixed by the crank arm, whereas the frequency could be controlled by a change in the voltage across the motor. To cover the range of nondimensional parameters of interest, the shaker was operated at frequencies of 0.2–1.0 Hz. Figure 2 shows the blades and the shaker with a typical set of vortices visualized by dye streams.

Vortex Visualization

Tip vortices were visualized through dye injection near the leading edge at the blade tips. The objective was to illuminate the cross sections of the vortex filaments with a light sheet and record their motion from the window downstream of the test section. The Injection of diluted milk into the vortex cores proved to be the most suitable method of visualizing their motion. When viewed through the downstream window of the tunnel, this technique produced two bright spots, representing the cores, against the dark background, that could be recorded on digital videotape. The very details of visualization of the filaments are outlined in Refs. 25 and 27, but a typical video frame, showing the illuminated vortex cores, is presented in Fig. 3. The light sheet used for this purpose was a high-intensity white parallel beam perpendicular to the test section.

Video Data Acquisition

The vortex filament cross sections were illuminated with a high-intensity light sheet set perpendicularly to the test section. The motion of the vortex cores was recorded on a tape in Digital 8 format, at a rate of 30 frames/s, with a SONY® DCR-TRV110 Digital Video Camera Recorder. Each data set consisted of approximately 2 min, 20 s of data, corresponding to 4200 frames. This length was dictated by the amount of hard-drive space available on the computer used for this study. During a typical test case, after the vortex motion was recorded at a specific downstream location, a grid was introduced inside the water tunnel at the location of the light sheet. This grid of 2-by-2 in. squares was then recorded on tape for a few seconds to provide a scale for the vortex motion at this specific downstream location. The time history of the vortex core locations was then extracted from the video. Again, the reader is encouraged to consult Refs. 25 and 27 for further details.

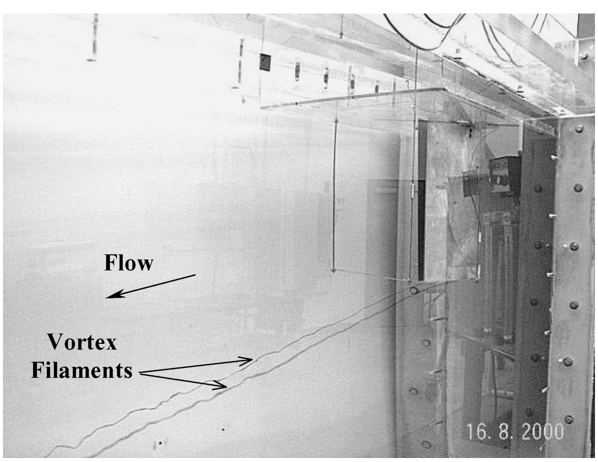
Results and Discussion

Several combinations of the experimental parameters were investigated. However, in the interest of brevity, only one of these cases will be discussed here. Table 1 shows the parameters used in this case, while additional results are presented in Ref. 27.

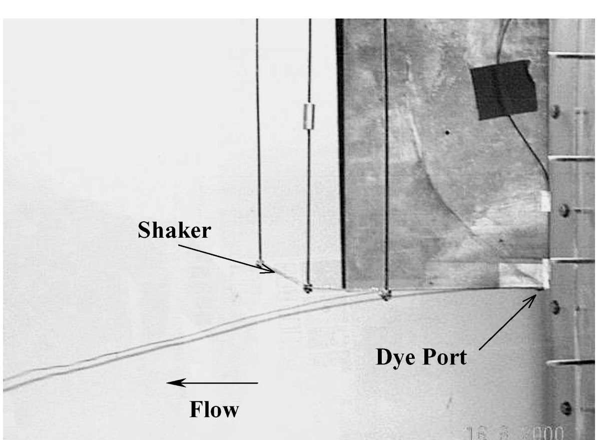
Data were recorded at prescribed distances downstream of the quarter-chord location of the blades. The downstream location closest to the blades was chosen to allow the wake enough time to roll up and the wingtip vortices to fully develop. Normally, the distance behind the wing, required for complete wake roll-up, is many times the wingspan. However, in the present cases the roll-up process took place rather quickly owing to the presence of separated boundary layers over a large part of the blades. The farthest location from the blades depended on how far downstream the dye would remain in the cores. Figure 4 shows the coordinates of a typical vortex cross

Table 1 Experimental parameters

Parameter	Value
Blade chord length, in.	8.1
Blade span, in.	11.0
Blade separation distance, in.	4.0
Angle of attack, deg	8.0
Flow speed, ft/s	0.25
Downstream locations, in.	29, 35, 41
Alcohol, milk, water ratio	1/12, 1/3, 7/12
Frequencies, Hz	0.2–1.0 (0.2)
Excitation amplitude, in.	0.10



a) Standard view



b) Close-up view

Fig. 2 Wide blades and shaker.

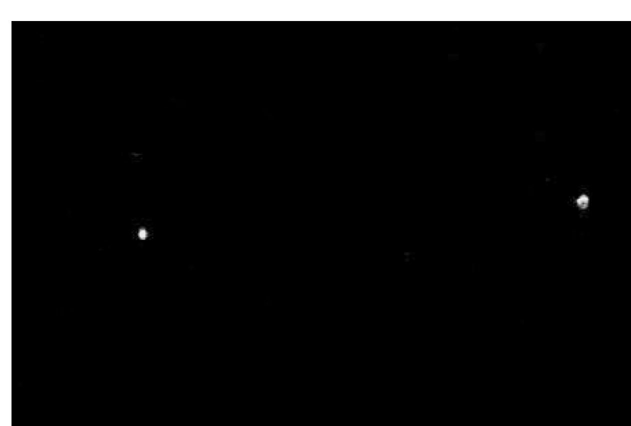


Fig. 3 Example video frame showing illuminated vortex cores.

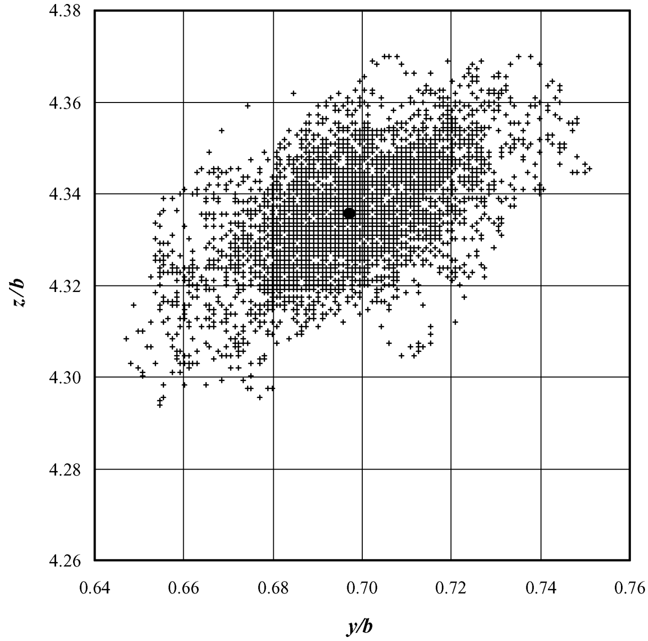


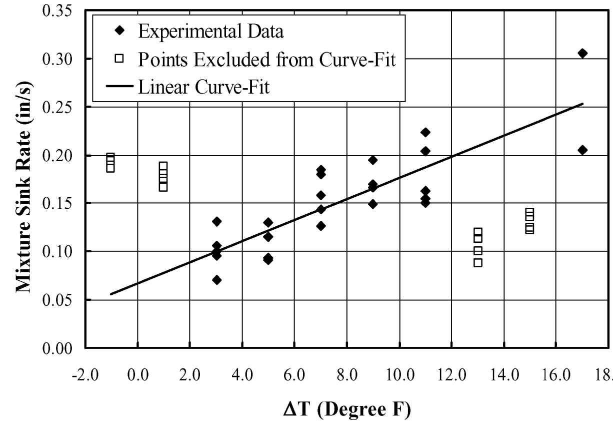
Fig. 4 Typical vortex position over time.

section at one of the downstream positions over time. The average vortex position was obtained from this type of data, along with the rms of the amplitude and the inclination angles of the planes in which the motion was dominant.

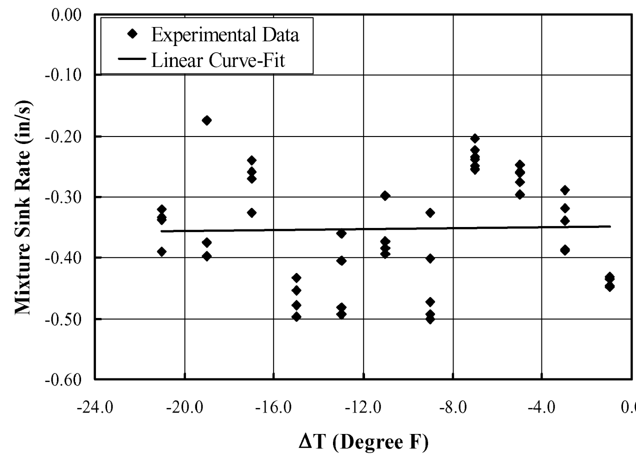
Initial experiments with a milk/water mixture of 1/3 by volume produced reasonable results. However, buoyancy was suspected of causing the mixture to leave the core and form spirals around the vortex filaments. The amplitude of this spiraling motion could be mistaken for the amplitude of vortex motion by the experimental method used here. A set of experiments was performed in which the mixture was released in the water tunnel horizontally, in the absence of the vortex filaments or water flow. The sink rate of the mixture was measured with a stopwatch. The results shown in Fig. 5a clearly indicated that the mixture was heavier than water. Also, as expected, the sink rate was related to the temperature difference between the mixture and the water. Addition of isopropyl alcohol (commercial rubbing alcohol) to the mixture proved to be an effective method of altering the buoyancy, as shown in Fig. 5b. The data scatter shown in Fig. 5b is primarily due to the higher diffusion rate of the alcohol. Based on these data, for neutral buoyancy, the optimum mixture ratio was determined to be roughly 1/3 milk, 7/12 water, and 1/12 alcohol and was used in all of the tests. These data also indicated the need for equalization of the mixture and the water tunnel temperatures during the tests. This was accomplished by routing the dye line through the water in the tunnel.

Although the blade separation distance determined the vortex span b, the two parameters were not exactly equal. This was due to the slight contraction of the wake and separation of the vortex filaments from the blades near the leading edge. Therefore, the vortex separation distance was determined from the experimental data. Figure 6 shows this parameter for the case discussed in this paper. Three observations could be made from Fig. 6. First, the vortex span remained fixed over the three downstream observation location points. This was indicative of the completion of the wake contraction upstream of these points. Second, it is obvious from Fig. 6 that vortex span was not affected by the excitation frequency. Finally, a comparison of the values in Fig. 6 with the blade separation distance shows that although the two parameters are close, they do differ slightly.

Figure 7 shows the vortex vertical position at different downstream locations and frequencies. In the interest of clarity, some of the frequencies have been omitted in Fig. 7. Three facts are evident in Fig. 7. First, all of the curves were nearly straight lines, which indicates that the downstream locations for data acquisition were sufficiently far from the blades. Second, the slope was nearly the



a) $\frac{1}{3}$ **milk,** $\frac{2}{3}$ **water**



b) $\frac{1}{6}$ **alcohol,** $\frac{1}{3}$ **milk,** $\frac{1}{2}$ **water**

Fig. 5 Mixture sink rate in the water tunnel.

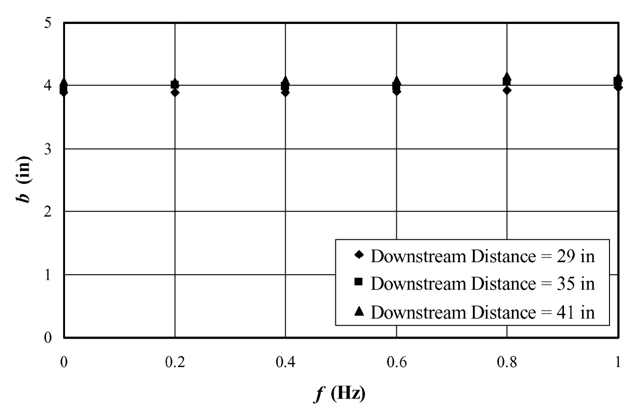


Fig. 6 Vortex span vs excitation frequency.

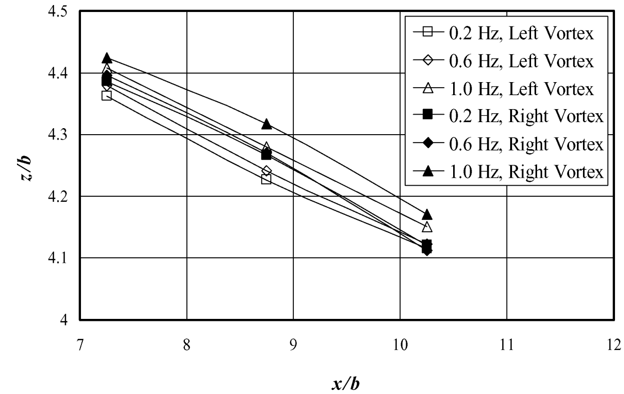


Fig. 7 Vortex vertical position.

Table 2 Experimentally measured circulation

f, Hz	β	Γ_{left}, ft^2/s	Γ_{right}, ft^2/s	Percent error
0.0	0.000	0.04565	0.04483	1.79
0.2	0.257	0.04305	0.04642	7.84
0.4	0.532	0.04305	0.04730	8.04
0.6	0.799	0.04485	0.04956	10.49
0.8	1.077	0.04106	0.04551	10.82
1.0	1.353	0.04498	0.04433	1.46

same for both vortex filaments, which indicates the near equality of the two vortex strengths. Finally, the slope that is directly related to the vortex strength was independent of the excitation frequency.

By the use of these two parameters, vortex strength Γ was determined from

$$\Gamma = 2\pi b w \tag{1}$$

where

$$w = V_\infty(\Delta z/\Delta x) \tag{2}$$

Table 2 shows the circulation strength that was determined for the present case by the use of the aforementioned method. The vortex strengths shown here are in good agreement with those of Rokhsaz et al.[24] that were obtained from hot-film anemometry. The values in Table 2 indicate that the estimated vortex strengths were not always the same for the two filaments. Also, the difference between the two vortex strengths was of the same order of magnitude as the variations in the strength of the same filament over the range of frequencies. This behavior was due to the method of aligning the blades. Vortex equality was ensured by alignment of the blades such that the two vortices would sink at the same rate from the blades to the end of the tunnel. However, the two filaments moved constantly. Therefore, during the set up of the apparatus, the average vortex vertical position had to be estimated visually. This inexact procedure resulted in vortex inequalities of the order of 10% in most cases.

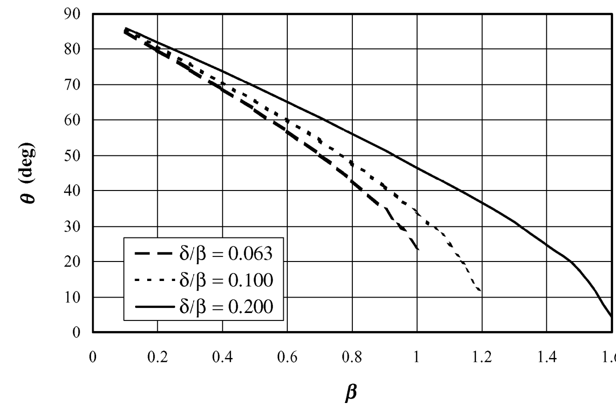
Until now, all indicators pointed to the integrity of the experimental data. The vortex span, obtained from the mean position of the vortex cores remained insensitive to downstream distance from the blades and the excitation frequency. The associated downwash appeared constant and repeatable, regardless of the excitation frequency, whereas the estimated vortex strengths remained within a 10% band. However, the inclination angles of the planes in which the vortex motion was dominant were not in complete agreement with calculations. Figure 8a shows the theoretically predicted angles,[6] whereas the experimentally obtained values for the present case are shown in Fig. 8b. The theory predicted a monotonic decrease in the inclination angle of the left vortex with increasing nondimensional frequency. The angles were predicted to vary from 90 to 25–45 deg, depending on the integration cutoff distance and the excitation frequency. However, the experimental values remained confined to a 25-deg band between 40 and 65 deg. Although these values were of correct magnitude, no specific trend relative to frequency or to downstream position could be discerned from these results.

Likewise, the growth in the amplitude of motion as characterized with the rms of the position indicated the correct behavior, as shown in Table 3. For the most part, the amplitude of motion increased with distance downstream of the blades, as well as with increasing excitation frequency. Both of these trends agreed with the theoretical results. However, the resulting amplitude growth rates, shown in Fig. 9, did not agree well with the theoretically predicted values.[6] In fact, Fig. 9 shows considerable scatter in the experimentally measured values of this parameter. Detailed examination of these results revealed two possible sources of error that have not been analyzed in any length at this writing.

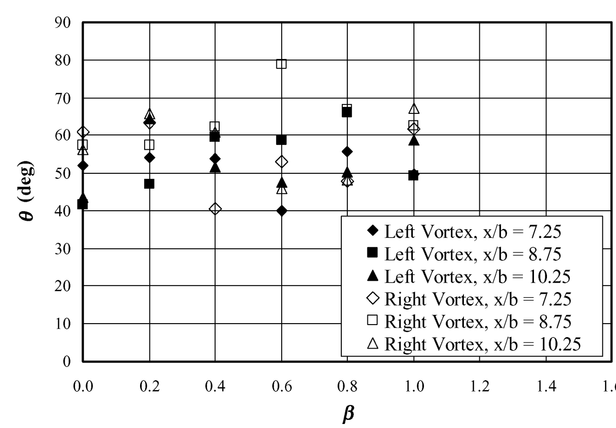
The first source of error was the excitation frequency. Fourier transforms of the time history of motion revealed the dominance of low frequencies in every case. Figure 10 shows a typical set of results obtained from Fourier transform of time history of the vertical position of the left vortex. These data were taken at the middle downstream distance at the frequency of 0.6 Hz. Although the excitation

Table 3 Vortex rms, inches, position in the plane of motion at x/b

f, Hz	7.25	8.75	10.25
	Left		
0.0	0.1064	0.1144	0.1464
0.2	0.1172	0.1202	0.1453
0.4	0.1027	0.1388	0.1675
0.6	0.1353	0.1594	0.1730
0.8	0.1541	0.1613	0.1880
1.0	0.1683	0.2328	0.2044
	Right		
0.0	0.1039	0.1444	0.1800
0.2	0.1263	0.1419	0.1605
0.4	0.1084	0.1216	0.1816
0.6	0.1409	0.1414	0.1752
0.8	0.1319	0.1647	0.1753
1.0	0.1444	0.2033	0.2297



a) Theoretical prediction for the left vortex



b) Experimentally measured

Fig. 8 Inclination angles of the planes of motion.

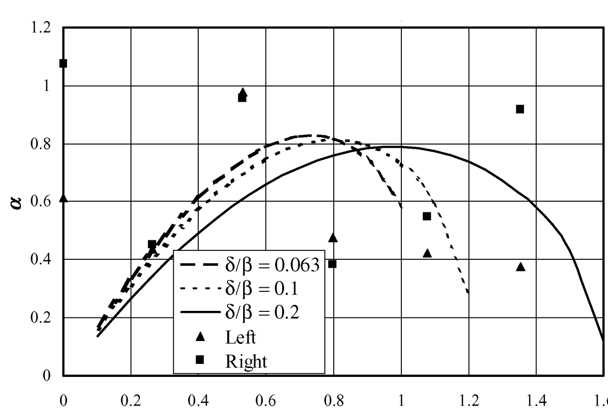


Fig. 9 Nondimensional amplitude growth rate.

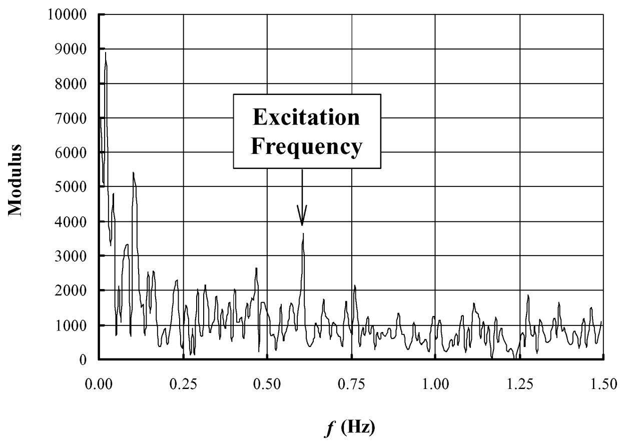


Fig. 10 Typical Fourier transform of vortex position time history.

frequency can be identified in Fig. 10, the strong influence of other frequencies is evident as well. This trend, which was present even in the absence of an excitation frequency, was entirely consistent with those reported by Rokhsaz et al.[25] for a single vortex and by Devenport et al.[26] for two vortex filaments. It is believed that the effects of these extraneous frequencies can be filtered out carefully to make the influence of the excitation frequency stand out more clearly.

The second source of error was the magnitude of the rms of the amplitude compared with the diameter of the mixture stream used to visualize the vortex. On the average, the latter quantity was approximately 0.2 in. in diameter, which was larger than the rms of the vortex position. This issue could be resolved by taking data at slightly farther distances downstream of the blades where the amplitude of motion has grown further.

The reader is cautioned that all of the results shown here were obtained with the raw experimental data. This data were not smoothed, conditioned, or filtered in any way because at the time of this writing, the authors do not understand the sources of error and their magnitudes sufficiently well to address this issue objectively. Even in this form, the results show all of the fundamental characteristics of Crow instability. The authors believe that the present data, despite their disagreements with the theoretical results, prove the integrity of the experimental approach. Also, it is believed that careful conditioning of the data to eliminate obvious sources of error could result in much better agreement with the theory.

Conclusions

Quantitative measurements related to Crow instability were made in a water tunnel by the use of a novel data acquisition technique that relied on flow visualization. The experimental approach was shown to be capable of estimating the vortex strength, the oscillation frequency of the vortex filaments, and the rate of growth of their amplitude simultaneously.

Results were presented for a pair of vortex filaments, typical of those of wing tips, created by individual vortex generating blades in a water tunnel. The filaments were excited over a range of frequencies and their responses, obtained from the time history of their positions, were presented. Vortex span and downwash were shown to be measured accurately. Vortex strengths, extracted from vortex span and downwash velocity, were shown to be consistent with those measured from hot-film anemometry. Other essential features of Crow instability, such as the inclination angle of the plane of vortex motion and the growth rate of the amplitude, were shown to be in qualitative agreement with the theoretical model. Disagreements between theoretical and experimental results were examined and discussed. These experimental results proved the viability of the approach and clearly demonstrated that the amplitude growth rate could be manipulated via the excitation frequency.

Acknowledgment

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